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# STRENGTH OF SHEET GLASS, SUPPORTED ALONG THE PERIMETER, UNDER A CONCENTRATED LOAD

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The results of experimental and theoretical studies of the behavior of sheet glass under a concentrated load are presented. A method and program for analyzing sheet glass when designing translucent structures, coatings and overlays have been developed.

***Key words:*** sheet glass, strength calculation, coating and overlay, concentrated load, computational program.

In recent years sheet glass has been used not only as a translucent material capable of transmitting light but also as building material capable of withstanding considerable mechanical loads. Sheet-glass building designs, coatings and overlays have appeared. In some buildings and structures, aside from a uniformly distributed load, a concentrated load acts on the sheet glass, as a result of which such designs must withstand concentrated loads.

Sheet glass in translucent structures can be represented as a plate which is supported along the perimeter. The strength and dimensional instability of the plates under a concentrated load were examined in [1, 2], but glass plates were not considered in these works. Considering this situation, the present author has performed experimental and theoretical studies of the behavior of sheet glass under a concentrated load.

## EXPERIMENTAL RESULTS

The stress-strain state and strength analysis for a plate of sheet-glass under a uniformly distributed load were presented in detail in [3]. The stress-strain state of a sheet-glass plate under a concentrated load is fundamentally different from the state produced by a distributed load.

The objective of the experimental studies was to study the actual strain distribution in the glass over the area of the plate, determine the sections with the highest stress, the position of the neutral axis and the load dependence of the deflection of the sample.

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The studies were performed on $1500 \times 1500$ mm samples of tempered glass ranging in thickness from 10 to 19 mm. The samples were supported in same manner as in translucent structures, i.e., between two rubber strips. A concentrated load was produced by a hydraulic jack and transmitted to the sample through a rubber strip on a 200 mm in diameter section (Fig. 1). Strain gauges with a 10 mm baseline were glued along the vertical axis and the diagonal of the sample in longitudinal and transverse directions on two opposite sides. In all, 64 sensors were placed on each sample. A TK 50 strain-measurement complex developed at the testing center was used as a secondary instrument. This complex makes it possible to measure strain in a material to accuracy $1 \times 10^{-6}$ relative strain units.

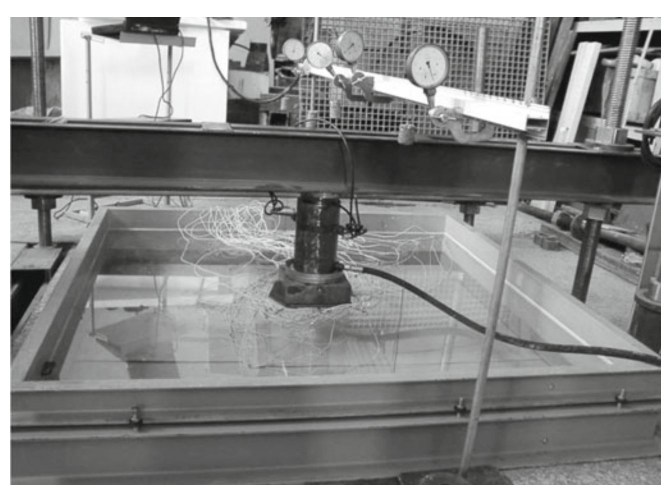

**Fig. 1.** Testing of sheet-glass samples.

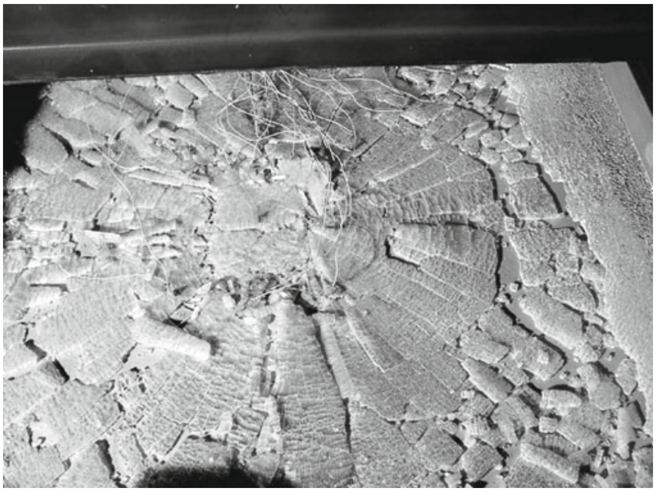

**Fig. 2.** Failure of a sample.

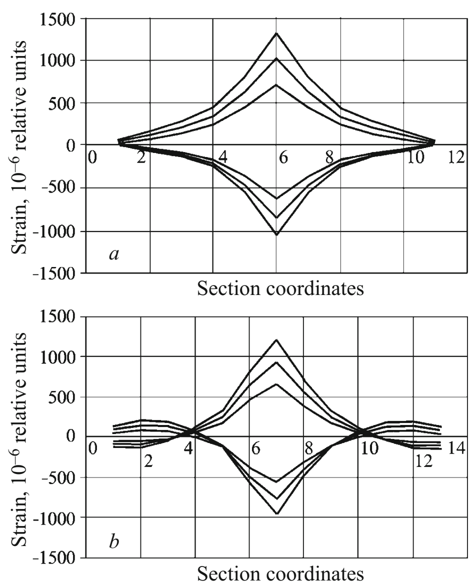


**Fig. 3.** Strain of glass in the test samples with $a/h = 80$ at different stress levels along the $y$ axis ($a$) and diagonal ($b$).

PAO 6 deflection meters were used to measure the deflection of the samples.

The load was produced in 5-min steps. The indications of the instruments were recorded during the holding periods. Failure of the samples was instantaneous. When it failed the glass broke into approximately 20 mm fragments (Fig. 2).

Analysis of the strain measurements showed that the highest strains along the $x$, $y$ axes and diagonal are observed at the center of the sample. But the maximum strains and therefore stresses are attained in the longitudinal direction along the $x$, $y$ axes immediately before failure (Fig. 3).

The position of the neutral axis in a transverse section depends on the ratio of the longitudinal edge length $a$ to the thickness $h$ of the sample ($\zeta = a/h$) and depends weakly on

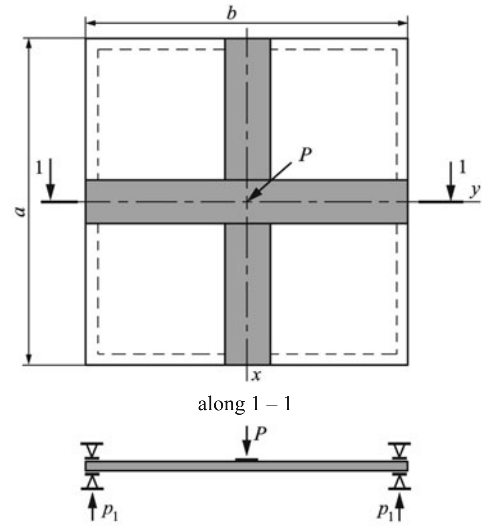


**Fig. 4.** Plate under a concentrated force.

the load. In thin plates with $\zeta > 120$ the neutral axis extends beyond the thickness of the sample and the entire section becomes elongated.

## COMPUTATIONAL RESULTS

Using the test results, we shall now examine a computational scheme with a concentrated force $P$ acting at the center of a plate supported along the perimeter (Fig. 4). Since experiments established that the maximum strains arise at the center of the plate along the $x$, $y$ axes, only these sections will be considered in the subsequent studies.

Let us cut out a strip of unit width along the x axis at the center of the plate and examine its stress state. Under the action of the concentrated force $P$ the plate presses against the supports with a definite force. However, this force is nonuniform along the perimeter of the plate, which must be taken into account by introducing a corresponding factor (1.5). The supportive response of a strip of unit width will be

$$p_1 = (P/(2a + 2b)) \times 1.5,$$

where $a$ and $b$ are the lengths of the longitudinal and transverse sides of the plate, mm, respectively.

The external bending moment at the center of the strip is given by the expression

$$M_1 = \frac{p_1 a}{2}.$$

The moment due to the internal forces is given by

$$M_2 = \frac{\sigma_p h^2}{12\lambda}.$$

Equating the moments of the external and internal forces we obtain an equation for determining the tensile stresses at the center of the plate

$$\sigma_t = \frac{6\lambda p_1 ac}{h^2},$$

where $\sigma_t$ are the tensile stresses in the glass, MPa; the factor $\lambda$ takes account of the position of the central axis; $a$ is an edge length of the plate, mm; $h$ is the thickness of the plate, mm; and, the factor $c$ takes account of the change of the stiffness of the plate as a function of its length-to-thickness ratio ($\zeta = a/h$).

For comparatively thick plates, when $\zeta < 120$, the factor $c$ must be determined using the relation

$$c = \alpha_1 - \left(\frac{p_1 a}{D}\zeta^2\right)^{0.3},$$

where the coefficient $\alpha_1$ is a function of $\zeta$; $D = E^3/12(1-\mu^2)$; $E$ is the modulus of elasticity, MPa; and, $\mu$ is the coefficient of transverse strain.

In flexible plates, where $200 > \zeta > 120$, large membrane tensile stresses appear and this relation for the coefficient $c$ leads to large (> 30%) errors. In this case the following relation must be used:

$$c = \alpha_2 \Big/ \left(\frac{p_1 \alpha}{D}\zeta^2\right)^{0.3},$$

where the coefficient $\alpha_2$ is a function of $\zeta$.

These relations were used to analyze a test sample of sheet glass with edge-to-thickness ratio $\zeta$ from 80 to 190. The theoretical and experimental values of the stresses in the glass for different loads are compared in Fig. 5.

As is evident from the plots the agreement between the theoretical and experimental values of the stresses is good.

The deflection at the center of the plate can be determined from the relation

$$f = \beta\left(\frac{Pa^2}{D}\right)^{\gamma},$$

where $\beta = \dfrac{40}{(0.15\zeta)^{0.5}}$; $f$ is the deflection at the center of the plate, mm; $P$ is the concentrated force, kN; $a$, $b$ are the

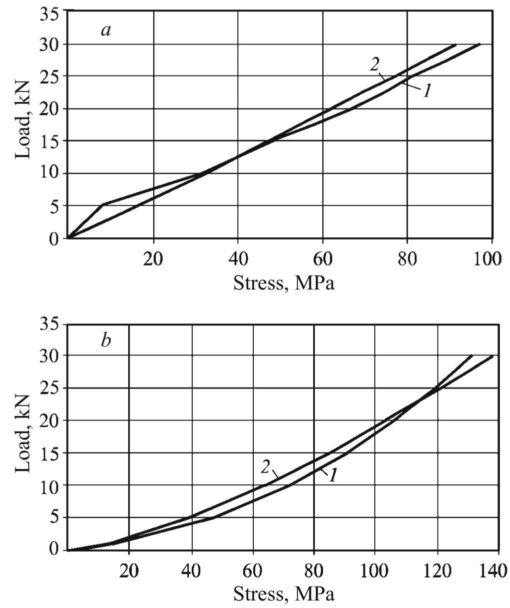


**Fig. 5.** Comparison of the experimental (*1*) and theoretical (*2*) values of the stresses in glass: *a*) $a/h = 80$; *b*) $a/h = 190$.

lengths of the long and short edges of the plate, mm; $D$ is the stiffness of the plate; coefficient $\gamma$ takes account of the change of the stiffness of the plate as a function of $\zeta$.

The results of the present studies were used to develop the Solid glass 2 computer program for calculating the strength of sheet glass under a concentrated load. This program is used to design translucent structures, coatings and overlays. To ensure structural safety the program permits introducing the appropriate safety factors taking account of the building's liability.

The relations presented above and the Solid glass 2 program can be used to analyze sheet-glass when designing translucent structures, coatings, and overlays.

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